In a nutshell: The wave equation in one dimension

Given a wave speed c, two spatial boundary points $[a, b]$ and two time boundaries $[t_0, t_f]$, and functions that give the boundary values at time t, $u_a(t)$ and $u_b(t)$ as well as an initial amplitude $u_0(x)$ and a rate of change of that amplitude $u_0^{(1)}(x)$ at time t_0.

Parameters:

- n_x The number of sub-intervals into which $[a, b]$ will be divided.
- n_t The number of sub-intervals into which time will be divided where $n_t \geq \left\lceil \frac{2c\left(t_f - t_0\right)}{h} \right\rceil$.

1. Set $h \leftarrow \frac{b-a}{n_x}$ and $x_k \leftarrow a + kh$ noting that $x_{n_x} = b$.
2. Set $\Delta t \leftarrow \frac{t_f - t_0}{n_t}$ and $t_\ell \leftarrow t_0 + \ell \Delta t$.
3. For k going from 0 to n_x, set $u_{k,0} \leftarrow u_{\text{init}}\left(x_k\right)$.
4. Set $u_{0,1} \leftarrow u_a\left(t_1\right)$ and $u_{n_x,1} \leftarrow u_b\left(t_1\right)$, and

   for k going from $n_x - 1$, set $u_{k,1} \leftarrow u_0\left(x_k\right) + u_0^{(1)}\left(x_k\right)\Delta t + \frac{1}{2}\left(c\Delta t\right)^2 \frac{u_{k-1,0} - 2u_{k,0} + u_{k+1,0}}{h^2}$.
5. For ℓ going from 1 to $n_t - 1$,
   a. Set $u_{0,\ell+1} \leftarrow u_a\left(t_{\ell+1}\right)$ and $u_{n_x,\ell+1} \leftarrow u_b\left(t_{\ell+1}\right)$, and
   b. for k going from 1 to $n_x - 1$, set $u_{k,\ell+1} \leftarrow 2u_{k,\ell} - u_{k,\ell-1} + \left(c\Delta t\right)^2 \frac{u_{k-1,\ell} - 2u_{k,\ell} + u_{k+1,\ell}}{h^2}$.